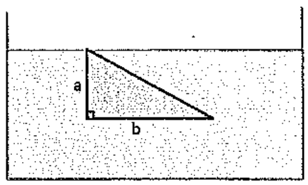
SOLUTION

CALCULUS II **QUIZ IV** MATH-UA.0122-007

Write your solutions in steps.

1. (4 points) Find the length of the curve $y = \ln \sec x$, $0 \le x \le \frac{\pi}{4}$.
2. (3 points) An aquarium with length 1, width b and depth c is full of water. Find the work needed to pump half of water out of the aquarium. (Density of water is ρ, and gravitational acceleration is g)
3. (3 points) A vertical triangular plate is submerged in water as shown in the picture. Compute the hydrostatic force against one side of the plate. (Density of water is ρ, and gravitational acceleration is g)



1.

$$y' = \frac{1}{\sec x} \cdot (\sec x)'$$

$$= \cos x \cdot \left(\frac{1}{\cos x}\right)'$$

$$= \cos x \cdot \frac{\sin x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x} = \tan x$$

The arc length is

$$\int_0^{\frac{\pi}{4}} \sqrt{1+(y')^2}\, dx = \int_0^{\frac{\pi}{4}} \sqrt{1+\tan^2 x}\, dx$$

$$= \int_0^{\frac{\pi}{4}} \sec x\, dx$$

$$= \ln(1+\sqrt{2})$$

2. Divide the upper half of the pool into n horizontal slices of depth $\Delta z = \frac{\frac{c}{2}}{n}$

The volume of the i-th piece is

$1 \times b \times \Delta z = b\Delta z$

the mass is $\rho b \Delta z$

when Δz is small, we approximate the work done on this piece by

$(\rho b \Delta z) g z_i^*$,

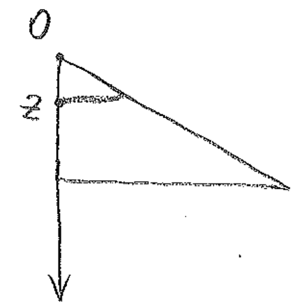
where $z_i^* \in [z_{i-1}, z_i]$

Then take the limit we get the acture work:

$$\lim_{\Delta z \to 0} \rho b \Delta z g z_i^* = \int_0^{\frac{c}{2}} \rho b g z\, dz$$

$$= \frac{\rho b g}{2} z^2 \Big|_0^{\frac{c}{2}} = \frac{\rho g b c^2}{8}$$

3. At depth z, the horizontal section is of length $\frac{bz}{a}$.



So the hydrostatic force is

$$\int_0^a \rho g z \cdot \frac{bz}{a} dz = \frac{\rho g b}{a} \int_0^a z^2 dz = \frac{\rho g a^2 b}{3}$$